

Name: _____

Solutions

This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assignment (that is answer all problems). I will then select 20 problems at random to grade, each worth 3 points each. The space left between each question is indicative of how much work you should show. If there are any problems you find particularly difficult, circle them in red. If there are any particular questions you would like feedback on, circle them in green. These are examples of questions that might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them without it.

1. Simplify each of the following expressions.

$$(a) \frac{x^3 + 10x^2}{x^2 + 6x - 40} = \frac{x^2(x+10)}{(x+10)(x-4)}$$

$$\text{Answer: } \frac{x^2}{x-4}$$

$$(d) \frac{6x^2 + 13x + 5}{3x^2 + 26x + 35} = \frac{(3x+5)(2x+1)}{(3x+5)(x+7)}$$

$$\text{Answer: } \frac{2x+1}{x+7}$$

$$(b) \frac{x^2 + 18x + 72}{2x^2 + 11x - 6} = \frac{(x+6)(x+12)}{(2x-1)(x+6)}$$

$$\text{Answer: } \frac{x+12}{2x-1}$$

$$(e) \frac{-x^2 + 10x - 9}{-x^2 + 6x + 27} = \frac{x^2 - 10x + 9}{x^2 - 6x - 27} = \frac{(x-1)(x-9)}{(x+3)(x-9)}$$

$$\text{Answer: } \frac{x-1}{x+3}$$

$$(c) \frac{x^2 - 3x - 28}{49 - x^2} = \frac{(x-7)(x+4)}{(x-7)(x+7)}$$

$$\text{Answer: } \frac{x+4}{x+7}$$

$$(f) \frac{x^3 + x^2 - 20x}{x^4 - 12x^3 + 36x^2} = \frac{x(x+5)(x-4)}{x^2(x-6)^2}$$

$$\text{Answer: } \frac{(x+5)(x-4)}{x(x-6)^2}$$

2. Perform the indicated operation and simplify.

$$(a) \frac{x^2 + 14x + 40}{x^2 + 2x - 8} \cdot \frac{x^2 + 5x - 14}{x^2 + 7x - 30} = \frac{(x+4)(x+10)}{(x+4)(x-2)} \cdot \frac{(x-2)(x+7)}{(x+10)(x-3)}$$

Answer: $\frac{x+7}{x-3}$

$$(b) \frac{4x^3 - x^2 - 3x}{x^2 - 10x + 25} \cdot \frac{10 + 3x - x^2}{x^4 - x^3} = \frac{x(4x+3)(x-1)}{(x-5)^2} \cdot \frac{(5-x)(x+2)}{x^3(x-1)}$$

Answer: $\frac{-(4x+3)(x+2)}{x^2(x-5)}$

$$(c) \frac{x^2 + 5x - 24}{x^2 - 5x + 4} \div \frac{x^2 + x - 12}{x - 1} = \frac{(x+8)(x-3)}{(x-1)(x-4)} \cdot \frac{(x-1)}{(x+4)(x-3)}$$

Answer: $\frac{x+8}{(x-4)(x+4)}$

$$(d) \frac{6x^2 + x^3 - x^4}{x^2 - 4} \div \frac{3x^3 - 9x^2}{x^2 + 6x - 16} = \frac{-x^2(x-3)(x+2)}{(x+2)(x-2)} \cdot \frac{(x+8)(x-2)}{3x^2(x-3)}$$

Answer: $\frac{-(x+8)}{3}$

$$(e) \frac{3x^2 + 23x + 14}{x^2 + 4x + 3} \div \frac{6x^2 + 13x + 6}{x^2 + 2x + 1} = \frac{(3x+2)(x+7)}{(x+1)(x+3)} \cdot \frac{(x+1)^2}{(3x+2)(2x+3)}$$

Answer: $\frac{(x+7)(x+1)}{(x+3)(2x+3)}$

$$(f) \frac{2}{x+4} \div \frac{6x^3 + 17x^2}{x^2 + 3x - 4} = \frac{2}{x+4} \cdot \frac{(x+4)(x-1)}{x^2(6x+17)}$$

Answer: $\frac{2(x-1)}{x^2(6x+17)}$

$$(g) 5x^2 - 18x - 8 \div \frac{x-4}{x+6} = (5x+2)(x-4) \cdot \frac{(x+6)}{(x-4)}$$

Answer: $(5x+2)(x+6)$

3. Perform the indicated operation and simplify.

$$(a) \frac{2x}{x+9} - \frac{x-1}{x} = \frac{2x \cdot x - (x-1)(x+9)}{x(x+9)}$$

$$= \frac{2x^2 - (x^2 + 8x - 9)}{x(x+9)}$$

$$\frac{x^2 - 8x + 9}{x(x+9)}$$

Answer: _____

$$(b) \frac{x+1}{x-1} + \frac{6}{x-7} = \frac{(x+1)(x-7) + 6(x-1)}{(x-1)(x-7)}$$

$$= \frac{x^2 - 6x - 7 + 6x - 6}{(x-1)(x-7)}$$

$$\frac{x^2 - 13}{(x-1)(x-7)}$$

Answer: _____

$$(c) \frac{9}{x^2-4} - \frac{7x}{x^2-4x+4} = \frac{9}{(x-2)(x+2)} - \frac{7x}{(x-2)^2}$$

$$= \frac{9(x-2) - 7x(x+2)}{(x+2)(x-2)^2}$$

$$= \frac{9x - 18 - 7x^2 - 14x}{(x+2)(x-2)^2} - \frac{(7x^2 + 5x + 18)}{(x+2)(x-2)^2}$$

Answer: _____

$$(d) \frac{3}{6x-x^2} - \frac{x}{x^2-5x-6} = \frac{3}{x(6-x)} + \frac{x}{(6-x)(x+1)}$$

$$= \frac{3(x+1) + x(x)}{x(6-x)(x+1)}$$

$$\frac{x^2 + 3x + 1}{x(6-x)(x+1)}$$

Answer: _____

$$\begin{aligned}
 \text{(e)} \quad & \frac{2}{x^2 - 4x - 12} + \frac{8x}{x^2 + 12x + 20} \\
 &= \frac{2}{(x-6)(x+2)} + \frac{8x}{(x+2)(x+10)} \\
 &= \frac{2(x+10) + 8x(x-6)}{(x-6)(x+2)(x+10)} \\
 &= \frac{2x+20 + 8x^2 - 48x}{(x-6)(x+2)(x+10)} \\
 &= \frac{8x^2 - 46x + 20}{(x-6)(x+2)(x+10)} \\
 &= \frac{2(4x^2 - 23x + 10)}{(x-6)(x+2)(x+10)}
 \end{aligned}$$

Answer: _____

$$\begin{aligned}
 \text{(f)} \quad & \frac{2}{3x^2} - \frac{1}{4x^7} + \frac{7}{6x^3} \\
 &= \frac{2}{3x^2} - \frac{1}{2^2x^7} + \frac{7}{2 \cdot 3x^3} \\
 &= \frac{2 \cdot 2^2 \cdot x^5 - 3 + 7 \cdot 2 \cdot x^4}{3 \cdot 2^2 \cdot x^7} \\
 &= \frac{8x^5 - 3 + 14x^4}{12x^7}
 \end{aligned}$$

Answer: _____

$$\begin{aligned}
 \text{(g)} \quad & \frac{2x+1}{4x^2 - 3x - 7} - \frac{x+3}{x+1} + \frac{x}{4x-7} \\
 &= \frac{2x+1}{(x+1)(4x-7)} - \frac{x+3}{x+1} + \frac{x}{4x-7} \\
 &= \frac{2x+1 - (x+3)(4x-7) + x(x+1)}{(x+1)(4x-7)} \\
 &= \frac{2x+1 - (4x^2 + 5x - 21) + x^2 + x}{(x+1)(4x-7)} \\
 &= \frac{-3x^2 - 2x + 22}{(x+1)(4x-7)}
 \end{aligned}$$

Answer: _____

$$\begin{aligned}
 \text{(h)} \quad & \frac{3}{x^2} + \frac{x+9}{x^2 + 5x} - \frac{2}{x^2 + 10x + 25} \\
 &= \frac{3}{x^2} + \frac{x+9}{x(x+5)} - \frac{2}{(x+5)^2} \\
 &= \frac{3(x+5)^2 + (x+9)x(x+5) - 2x^2}{x^2(x+5)^2} \\
 &= \frac{x^3 + 15x^2 + 75x + 75}{x^2(x+5)^2}
 \end{aligned}$$

Answer: _____

4. Rationalise the denominator for each of the following expressions. You may assume all letters represent positive numbers.

(a) $\frac{2}{4-\sqrt{x}} \cdot \frac{(4+\sqrt{x})}{(4+\sqrt{x})}$

Answer: $\frac{2(4+\sqrt{x})}{16-x}$

(f) $\frac{9}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$

Answer: $\frac{9\sqrt{y}}{y}$

(b) $\frac{9}{(\sqrt{3y}+2)} \cdot \frac{(\sqrt{3y}-2)}{(\sqrt{3y}-2)}$

Answer: $\frac{9(\sqrt{3y}-2)}{3y-4}$

(g) $\frac{3}{7x}$ oops

Answer: $\frac{3}{7x}$

(c) $\frac{4}{(\sqrt{7}-6\sqrt{x})} \cdot \frac{(\sqrt{7}+6\sqrt{x})}{(\sqrt{7}+6\sqrt{x})}$

Answer: $\frac{4(\sqrt{7}+6\sqrt{x})}{7-36x}$

(h) $\frac{1}{\sqrt[4]{x}} \cdot \frac{(\sqrt[4]{x})^3}{(\sqrt[4]{x})^3}$

(i) $\frac{12}{\sqrt[5]{3x^2}} \cdot \frac{(\sqrt[5]{3x^2})^4}{(\sqrt[5]{3x^2})^4} = \frac{12\sqrt[5]{3x^2}^4}{3x^2} = \frac{4\sqrt[5]{81x^8}}{x^2} = \frac{4\sqrt[5]{81x^3} \cdot x}{x^2}$
 Answer: $\frac{4\sqrt[5]{81x^3}}{x}$

(d) $\frac{-6}{(\sqrt{5x}+10\sqrt{y})} \cdot \frac{(\sqrt{5x}-10\sqrt{y})}{(\sqrt{5x}-10\sqrt{y})}$

Answer: $\frac{-6(\sqrt{5x}-10\sqrt{y})}{5(x-20y)}$

(e) $\frac{x+4}{(x-\sqrt{x})} \cdot \frac{(x+\sqrt{x})}{(x+\sqrt{x})}$

Answer: $\frac{(x+4)(x+\sqrt{x})}{x(x-1)}$

(j) $\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{(\sqrt{5}-\sqrt{3})^2}{5-3}$

$= \frac{5-2\sqrt{15}+3}{2}$

Answer: $4-\sqrt{15}$

5. Perform long division on the following expressions.

(a) Divide $7x^2 + 4x - 9$ by $x - 1$.

$$\begin{array}{r} 7x + 11 \\ x-1 \overline{) 7x^2 + 4x - 9} \\ \underline{-(7x^2 - 7x)} \\ 11x - 9 \\ \underline{-(11x - 11)} \\ 2 \end{array}$$

Answer: $\underline{7x + 11 + \frac{2}{x-1}}$

(c) Divide $x^4 - 2x^2 + 7x$ by $x - 4$.

$$\begin{array}{r} x^3 + 4x^2 + 14x + 63 \\ x-4 \overline{) x^4 - 2x^2 + 7x} \\ \underline{-(x^4 - 4x^3)} \\ 4x^3 - 2x^2 + 7x \\ \underline{-(4x^3 - 16x^2)} \\ 14x^2 + 7x \\ \underline{-(14x^2 - 56x)} \\ 63x \\ \underline{-(63x - 252)} \\ 252 \end{array}$$

Answer: $\underline{x^3 + 4x^2 + 14x + 63 + \frac{252}{x-4}}$

(b) Divide $8x^3 - 4x + 1$ by $x + 6$.

$$\begin{array}{r} 8x^2 - 48x + 284 \\ x+6 \overline{) 8x^3 - 4x + 1} \\ \underline{-(8x^3 + 48x^2)} \\ -48x^2 - 4x + 1 \\ \underline{-(-48x^2 - 288x)} \\ 284x + 1 \\ \underline{-(284x + 1704)} \\ -1703 \end{array}$$

Answer: $\underline{8x^2 - 48x + 284 - \frac{1703}{x+6}}$

(d) Divide $2x^4 - 9x^3 + 2x + 8$ by $x + 3$.

$$\begin{array}{r} 2x^3 - 15x^2 - 45x + 137 \\ x+3 \overline{) 2x^4 - 9x^3 + 2x + 8} \\ \underline{-(2x^4 + 6x^3)} \\ -15x^3 + 2x + 8 \\ \underline{-(-15x^3 - 45x^2)} \\ -45x^2 + 2x + 8 \\ \underline{-(-45x^2 - 135x)} \\ 137x + 8 \\ \underline{-(137x + 411)} \\ -403 \end{array}$$

Answer: $\underline{2x^3 - 15x^2 - 45x + 137 - \frac{403}{x+3}}$
Cont.

(e) Divide $8x^4 + x^3 - 3x^2 + 1$ by $x^2 - 2$.

$$\begin{array}{r}
 8x^2 + x + 3 \\
 x^2 - 2 \overline{) 8x^4 + x^3 - 3x^2 + 1} \\
 \underline{-(8x^4 - 6x^2)} \\
 x^3 + 3x^2 \\
 \underline{-(x^3 - 2x)} \\
 3x^2 + 2x + 1 \\
 \underline{-(3x^2 - 6)} \\
 2x + 7
 \end{array}$$

Answer: $\frac{8x^2 + x + 3}{x^2 - 2} + \frac{2x - 7}{x^2 - 2}$

(f) Divide $2x^2 - 17x - 38$ by $2x + 3$.

$$\begin{array}{r}
 x - 10 \\
 2x + 3 \overline{) 2x^2 - 17x - 38} \\
 \underline{-(2x^2 + 3x)} \\
 -20x - 38 \\
 \underline{-(-20x - 30)} \\
 -8
 \end{array}$$

Answer: $x - 10 - \frac{8}{2x + 3}$

(i) Divide $4x^5 - 7x^3 + x^2 - 4x + 2$ by $2x^2 - 3x - 6$.

$$\begin{array}{r}
 2x^3 + 3x^2 + 7x + 20 \\
 2x^2 - 3x - 6 \overline{) 4x^5 - 7x^3 + x^2 - 4x + 2} \\
 \underline{-(4x^5 - 6x^4 - 12x^3)} \\
 6x^4 + 5x^3 + x^2 - 4x + 2 \\
 \underline{-(6x^4 - 9x^3 - 18x^2)} \\
 15x^3 + 11x^2 - 4x + 2
 \end{array}$$

(g) Divide $42x^2 - 33$ by $7x + 7$.

$$\begin{array}{r}
 6x - 6 \\
 7x + 7 \overline{) 42x^2 - 33} \\
 \underline{-(42x^2 + 42x)} \\
 -42x - 33 \\
 \underline{-(-42x - 42)} \\
 9
 \end{array}$$

Answer: $6x - 6 + \frac{9}{7x + 7}$

(h) Divide $x^4 + 4x^3 + x - 1$ by $x^2 + 3x - 5$.

$$\begin{array}{r}
 x^2 + x + 2 \\
 x^2 + 3x - 5 \overline{) x^4 + 4x^3 + x - 1} \\
 \underline{-(x^4 + 3x^3 - 5x^2)} \\
 x^3 + 5x^2 + x - 1 \\
 \underline{-(x^3 + 3x^2 - 5x)} \\
 2x^2 + 6x - 1 \\
 \underline{-(2x^2 + 6x - 10)} \\
 9
 \end{array}$$

Answer: $x^2 + x + 2 + \frac{9}{x^2 + 3x - 5}$

$$\begin{array}{r}
 14x^3 + 19x^2 - 4x + 2 \\
 \underline{-(14x^3 - 21x^2 - 42x)} \\
 40x^2 + 38x + 2 \\
 \underline{-(40x^2 - 60x - 120)} \\
 98x + 122
 \end{array}$$

Answer: $2x^3 + 3x^2 + 7x + 20 + \frac{98x + 122}{2x^2 - 3x - 6}$